Manual for SSPACK: the semiseparable software package

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today

Abstract

The manual briefly explains the semiseparable software package as presented on the webpage `http://www.cs.kuleuven.ac.be/~raf`. The publications connected to the software [7, 3, 6, 8, 9, 4, 5], other related publications of the same authors [1, 2].

1 Software package for semiseparable matrices

This paper gives an overview of the routines that are included in the software package for semiseparable matrices. The software is written for the programming environment MATLAB. The package can be downloaded from `http://www.cs.kuleuven.ac.be/~marc/software/.`

The tools contained in the software package are not always restricted to the implementations, as described in the thesis. Several of the implementations were adapted, such that they are applicable to a larger class of matrices, e.g. to unsymmetric matrices, lower triangular semiseparable matrices etc.

The tools described here are designed for, or working with: symmetric or unsymmetric **S**emi**S**eparable matrices (**SS**), **U**pper triangular **S**emiseparable matrices (**US**), **L**ower triangular **S**emiseparable matrices (**LS**) and **H**essenberg-**L**ike matrices (**HL**).

The given descriptions are very short. More help can be found when considering these routines in MATLAB and calling the help information of the function, e.g.

```
> help CSS;
```

will give much more detailed information about the routine **CSS**.

2 General tools

2.1 Retrieving the Givens-vector representation

- **BRSS:** ***B****uild the* ***R****epresentation of a* ***S****emi****S****eparable matrix*
  The function will retrieve the Givens-vector representation from a semiseparable input matrix. The input matrix can either be symmetric or unsymmetric. For an unsymmetric the lower triangular part and the strictly upper triangular part will be represented by two sequences of Givens transformations and two vectors.

- **BRHL:** ***B****uild the* ***R****epresentation of a* ***H****essenberg-****L****ike matrix.*
  The function will retrieve the Givens-vector representation of a Hessenberg-like matrix. The output consists of a Givens-vector representation, representing the lower triangular part, and the strictly upper triangular part of the Hessenberg-like matrix.

- **BRUS & BRLS:** Similar as above for **US** and **LS** matrices.

2.2 Constructing the matrices from the representation

- **BSS:** ***B****uild a full* ***S****emi****S****eparable matrix.*
  Given the Givens-vector representation of either a symmetric or an unsymmetric matrix, the function will build a full semiseparable matrix from it.

- **BUS & BLS & BHL:** The same as above for **US**, **LS** and **HL** matrices.

2.3 Various tools

- **SR:** ***S****wap the* ***R****epresentation.*
  A semiseparable matrix can be represented with the Givens-vector representation from two directions. This function changes this direction.
- **MULSS:** ***MUL****tiply a vector with a* ***S****emi****S****eparable matrix.*
  Performs an order $O(n)$ matrix vector multiplication. The matrix can either be symmetric or unsymmetric, in the unsymmetric case two sequences of Givens-vector representations have to be given as input.
- **MULUS & MULLS & MULHL:** The same as above for **US**, **LS** and **HL** matrices.
- **DETSS:** *Calculate the* ***DET****erminant of a* ***S****emi****S****eparable matrix.*
  Given a symmetric or unsymmetric semiseparable matrix, represented with the Givens-vector representation, the function will calculate the determinant in $O(n)$.
- **DETUS & DETLS & DETHL:** The same as above for **US**, **LS** and **HL** matrices.

3 Reduction Algorithms

- **CSS: C***onstruction of a* **S***ymmetric* **S***emiseparable matrix.*
  The function reduces via orthogonal similarity transformations the symmetric input matrix into semiseparable form. The presented output is in the Givens-vector representation form.
- **CUS & CLS & CHL:** Similar as above for **US**, **LS** and **HL** matrices.

4 *QR*-tools

4.1 *QR*-solver

- **SOLVSSD:** ***SOL****ve a system with a* ***S****emi****S****eparable plus* ***D****iagonal matrix.*
  This routine solves a system of equations for which the coefficient matrix is of diagonal plus semiseparable form in $O(n)$ operations. The semiseparable part is represented using the Givens-vector representation.
- **SOLVHLD:*SOL****ve a system with a* ***H****essenberg-****L****ike plus* ***D****iagonal matrix.*

4.2 Reduction to unreduced form

- **REDSS** ***RED****uction of a* ***S****emi****S****eparable matrix to unreduced form.*
  This function transforms via orthogonal similarity transformations the semiseparable matrix to unreduced form.
- **REDUS & REDHL:** The same as above for **US** and **HL** matrices.

4.3 An implicit *QR*-step without shift

- **QRSS:** *A* ***QR****-step on a* ***S****emi****S****eparable matrix.*
  This routine performs a step of *QR* without shift on a semiseparable matrix represented with the Givens-vector representation.
- **QRUS & QRHL:** The same as above for **US** and **HL** matrices.

4.4 An implicit *QR*-step with shift

- **QRSSS:** *A **QR**-step with **S**hift on a **S**emi**S**eparable matrix.*
  This routine performs a step of *QR* with shift on a semiseparable matrix represented with the Givens-vector representation.
- **QRSSSD:** *A **QR**-step with **S**hift on a **S**emi**S**eparable plus **D**iagonal matrix.*
  This routine performs a step of *QR*-with shift on a semiseparable matrix represented with the Givens-vector representation.
- **QRSUS & QRSHL:** The same as above for **US** and **HL** matrices.

4.5 The eigen(singular)value decomposition

- **EIGSS:** *The **EIG**envalue decomposition of a **S**emi**S**eparable matrix.*
  This routine calculates the eigenvalue decomposition of a symmetric semiseparable matrix in the Givens-vector representation.
- **EIGSSD:** *The **EIG**envalue decomposition of a **S**emi**S**eparable plus **D**iagonal matrix.*
  This routine calculates the eigenvalue decomposition of a symmetric semiseparable plus diagonal matrix in the Givens-vector representation.
- **EIGHL:** The same as above for **HL** matrices.
- **SVDUS:** *The **SVD** of **U**pper triangular **S**emiseparable matrices.*
  This routine calculates the singular value decomposition of an upper triangular semiseparable matrix represented with the Givens-vector representation.

5 Eigenvalue and singular value tools

- **SSEIG:** ***S**emi**S**eparable **EIG**envalue decomposition.*
  This routine, calculates the eigenvalue decomposition of a symmetric matrix by first transforming the original matrix to a similar semiseparable one.
- **HLEIG:** ***H**essenberg-**L**ike **EIG**envalue decomposition.*
  This routine, calculates the eigenvalue decomposition of a matrix by first transforming the original matrix to a similar Hessenberg-like one.
- **USSVD:** ***U**pper triangular **S**emiseparable **SVD**.*
  This routine calculates the singular value decomposition of matrices, by first transforming them to an upper triangular semiseparable matrix.

References

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